FALL 2019: MATH 558 QUIZ 10 SOLUTIONS

Each question is worth 5 points.

1. Let $f(x) = x^2 + x + 1 \in \mathbb{Q}[x]$ and take $\alpha \in \mathbb{C}$ a root of $f(x)$. Describe a typical element in $\mathbb{Q}(\alpha)$ and then calculate the multiplicative inverse of $2 + \alpha$ as an element of $\mathbb{Q}(\alpha)$.

Solution. $\mathbb{Q}(\alpha)$ is the set of complex numbers of the form $a + b\alpha$, with $a, b \in \mathbb{Q}$.

To find the multiplicative inverse of $2+\alpha$: Via the division algorithm, we obtain $x^2+x+1 = (2+x)(x-1)+3$. Substituting $x = \alpha$ yields $0 = (2 + \alpha)(\alpha - 1) + 3$. Thus, $3 = (2 + \alpha)(1 - \alpha)$, so $1 = (2 + \alpha)(\frac{1}{3} - \frac{1}{3}\alpha)$. Thus, $(2 + \alpha)^{-1} = \frac{1}{3} - \frac{1}{3}\alpha$.

2. Use the Rational Root Test to determine if the polynomial $p(x) = 2x^3 + 5x + 4$ has a rational root. Is $p(x)$ irreducible over $\mathbb{Q}$? Justify your answer.

Solution. By the Rational Root test, the possible rational roots of $p(x)$ are $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$. SInce the coefficients of $p(x)$ are all positive, $p(x)$ cannot have a positive root. For the negative roots, one checks that $p(-\frac{1}{2}) \neq 0, p(-1) \neq 0, p(-2) \neq 0, p(-4) \neq 0$, so $p(x)$ has not rational roots. Since $p(x)$ has degree three, it must be irreducible over $\mathbb{Q}$.